BIG Ideas

• Select and use appropriate operations to solve problems and justify solutions.
• Make connections among various representations of a numerical relationship.
• Use graphs, tables, and algebraic representations to make predictions and solve problems.

Key Vocabulary

null or empty set (p. 426)
identity (p. 426)
inequality (p. 430)

Real-World Link

Capacity You can use an inequality to express the maximum number of people that can be held in the Radio City Music Hall in New York City.

Foldables Study Organizer

Equations and Inequalities Make this Foldable to help you organize notes on equations and inequalities. Begin with a plain sheet of 8 1/2 by 11” paper.

1 Fold in half lengthwise.
2 Fold in thirds and then fold each third in half.
3 Open. Cut one side along the folds to make tabs.
4 Label each tab with a lesson number as shown.
GET READY for Chapter 8

Diagnose Readiness You have two options for checking Prerequisite Skills.

Option 2
Take the Online Readiness Quiz at pre-alg.com.

Option 1
Take the Quick Check below. Refer to the Quick Review for help.

Quick Check

Solve each equation. Check your solution.
(Lesson 3-3)
1. \(2x + 5 = 13\)
2. \(4n - 3 = 5\)
3. \(16 = 8 + \frac{d}{3}\)
4. \(\frac{c}{4} + 3 = -9\)
5. \(-18 = 4b + 10\)

Find each sum or difference.
(Lessons 2-2 and 2-3)
8. \(-28 + (-16)\)
9. \(17 + (-25)\)
10. \(-13 + 24\)
11. \(36 + (-18)\)
12. \(31 - 48\)
13. \(-16 - 7\)
14. \(4 - (-12)\)
15. \(-23 - (-29)\)

Find each product or quotient.
(Lessons 2-4 and 2-5)
17. \(-6(8)\)
18. \(-3 \cdot 5\)
19. \(-6(-25)\)
20. \(2(-4)(-9)\)
21. \(64 ÷ (-32)\)
22. \(-15 ÷ 3\)
23. \(-12 ÷ (-3)\)
24. \(24 ÷ (-2)\)
25. CHEMISTRY A solution cooled at a rate of 6°F every 5 minutes. What was the change in temperature after \(\frac{1}{2}\) hour? (Lesson 2-5)

Example 1
Solve \(\frac{r}{4} + 6 = 5\).
\[
\frac{r}{4} + 6 = 5
\]
Write the equation.
\[
\frac{r}{4} = -1
\]
Subtract 6 from each side.
\[
4\left(\frac{r}{4}\right) = 4(-1)
\]
Undo division. Multiply each side by 4.
\[
r = -4
\]
Simplify.

Example 2
Find \(-30 - (-42)\).
\[
-30 - (-42) = -30 + 42
\]
To subtract \(-42\), add 42.
\[
= 12
\]
Simplify.

Example 3
Find \(6 \times (-15)\).
\(6 \times (-15) = -90\) The factors have different signs so the product is negative.
**Algebra Lab**

**Equations with Variables on Each Side**

In Chapter 3, you used algebra tiles and an equation mat to solve equations in which the variable was on only one side of the equation. You can use algebra tiles and an equation mat to solve equations with variables on each side of the equation.

**ACTIVITY 1**

The following example shows how to solve \( x + 3 = 2x + 1 \) using algebra tiles.

1. **Step 1** Model the equation.
   
   ![Diagram of algebra tiles showing \( x + 3 \) on the left side and \( 2x + 1 \) on the right side.

2. **Step 2** Remove the same number of \( x \)-tiles from each side of the mat until there is an \( x \)-tile by itself on one side.
   
   ![Diagram of algebra tiles with \( x = 2 \) on the right side, showing how to remove \( x \)-tiles.

3. **Step 3** Remove the same number of positive tiles from each side of the mat until the \( x \)-tile is by itself on one side.
   
   ![Diagram of algebra tiles showing the final solution \( x = 2 \).

There are two positive tiles on the left side of the mat and one \( x \)-tile on the right side. Therefore, \( x = 2 \). Since \( 2 + 3 = 2(2) + 1 \), the solution is correct.

**Analyze the Results**

Use algebra tiles to model and solve each equation.

1. \( 2x + 3 = x + 5 \)  
2. \( 3x + 4 = 2x + 8 \)  
3. \( 3x = x + 6 \)  
4. \( 6 + x = 4x \)  
5. \( 2x - 4 = x - 6 \)  
6. \( 5x - 1 = 4x - 5 \)  
7. Which property of equality allows you to remove a positive tile from each side of the mat?  
8. Explain why you can remove an \( x \)-tile from each side of the mat.
Some equations are solved by using zero pairs. Remember, you may add or subtract a zero pair from either side of an equation mat without changing its value. The following example shows how to solve $2x + 1 = x - 5$.

**ACTIVITY 2**

**Step 1** Model the equation.

![Equation mat showing $2x + 1 = x - 5$.]

**Step 2** Remove the same number of $x$-tiles from each side of the mat until there is an $x$-tile by itself on one side.

![Equation mat showing $2x - x + 1 = x - x + 5$.]

**Step 3** It is not possible to remove the same number of 1-tiles from each side of the mat. Add 1 negative tile to the left side to make a zero pair. Add 1 negative tile to the right side of the mat.

![Equation mat showing $x + 1 + (-1) = -5 + (-1)$.]

**Step 4** Remove the zero pair from the left side. There are 6 negative tiles on the right side of the mat.

![Equation mat showing $x = -6$.]

Therefore, $x = -6$. Since $2(-6) + 1 = -6 - 5$, the solution is correct.

**ANALYZE THE RESULTS**

Use algebra tiles to model and solve each equation.

9. $2x + 3 = x - 5$
10. $3x - 2 = x + 6$
11. $x - 1 = 3x + 7$
12. $x + 6 = 2x - 3$
13. $2x + 4 = 3x - 2$
14. $4x - 1 = 2x + 5$
15. Does it matter whether you remove $x$-tiles or 1-tiles first? Explain.
16. Explain how you could use models to solve $-2x + 5 = -x - 2$. 

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**Review Vocabulary**

**Zero Pair**
A pair of numbers that, when added together, equal zero.
*Example: $3 + (-3) = 0$*
Main Idea
- Solve equations with variables on each side.

Each bag on the balance contains the same number of blocks. (Assume that the paper bag weighs nothing.)

a. The two sides balance. Without looking in a bag, how can you determine the number of blocks in each bag?

b. Explain why your method works.

c. Suppose \( x \) represents the number of blocks in the bag. Write an equation that is modeled by the balance.

d. Explain how you could solve the equation.

Equations with Variables on Each Side To solve equations with variables on each side, use the Addition or Subtraction Property of Equality to write an equivalent equation with the variables on one side. Then solve the equation.

**EXAMPLE**

Solve \( 2x + 3 = 3x \). Check your solution.

\[
\begin{align*}
2x + 3 &= 3x & \text{Write the equation.} \\
2x - 2x + 3 &= 3x - 2x & \text{Subtract 2x from each side.} \\
3 &= x & \text{Simplify.}
\end{align*}
\]

To check your solution, replace \( x \) with 3 in the original equation.

**CHECK**

\[
\begin{align*}
2x + 3 &= 3x & \text{Write the equation.} \\
2(3) + 3 &= 3(3) & \text{Replace } x \text{ with 3.} \\
6 + 3 &= 9 & \text{Simplify.} \\
9 &= 9 & \text{The statement is true.}
\end{align*}
\]

The solution is 3.

1. Solve \( 7x = 5x + 4 \). Check your solution.
2. Solve each equation. Check your solution.

a. \(5x + 4 = 3x - 2\)

\[5x + 4 = 3x - 2\] Write the equation.

\[5x - 3x + 4 = 3x - 3x - 2\] Subtract 3x from each side.

\[2x + 4 = -2\] Simplify.

\[2x + 4 - 4 = -2 - 4\] Subtract 4 from each side.

\[2x = -6\] Simplify.

\[x = -3\] Check your solution.

b. \(2.4 + a = 2.5a - 4.5\)

\[2.4 + a = 2.5a - 4.5\] Write the equation.

\[2.4 + a - a = 2.5a - a - 4.5\] Subtract \(a\) from each side.

\[2.4 = 1.5a - 4.5\] Simplify.

\[2.4 + 4.5 = 1.5a - 4.5 + 4.5\] Add 4.5 to each side.

\[6.9 = 1.5a\] Simplify.

\[\frac{6.9}{1.5} = \frac{1.5a}{1.5}\] Divide each side by 1.5.

\[a = 4.6\] Check your solution.

2A. \(2x + 3 = 3x - 2\)  
2B. \(3.2 + 0.3x = 0.2x + 1.4\)

RENTALS Under Plan A, an annual membership costs $30 plus $1.50 for each DVD rental. Under Plan B, the annual membership costs $12 plus $3 for each DVD rental. What number of DVD rentals results in the same yearly cost?

Let \(v\) represent the number of videos rented.

\[30 + 1.50v = 12 + 3v\] Write an equation.

\[30 + 1.50v - 1.5v = 12 + 3v - 1.5v\] Subtract 1.5v from each side.

\[30 = 12 + 1.5v\] Simplify.

\[30 - 12 = 12 - 12 + 1.5v\] Subtract 12 from each side.

\[18 = 1.5v\] Simplify.

\[\frac{18}{1.5} = \frac{1.5v}{1.5}\] Divide each side by 1.5.

\[12 = v\] Simplify.

The yearly cost is the same for 12 rentals.

3. CRUISES Red Bird Cruises charges $85 per day plus a one-time fee of $75 for taxes and gratuities. King Cruises charges $100 per day plus a fee of $30. For what number of days do the cruise companies charge the same?
Solve each equation. Check your solution.

1. $4x - 8 = 5x$
2. $4x + 9 = 7x$
3. $12x = 2x + 40$
4. $6a = 26 + 4a$
5. $4x - 1 = 3x + 2$
6. $4k + 24 = 6k - 10$
7. $7.2 - 3c = 2c - 2$
8. $3 - 3.7b = 10.3b + 10$

9. **CAR RENTAL** Suppose you can rent a car from ABC Auto for either $25 a day plus $0.45 a mile or for $40 a day plus $0.25 a mile. What number of miles results in the same cost for one day?

Define a variable and write an equation to find each number. Then solve.

10. Twice a number is 220 less than six times the number. What is the number?
11. Fourteen less than three times a number equals the number. Find the number.

12. **GEOGRAPHY** South Carolina's coastline is 358 kilometers longer than twice the coastline of North Carolina. It is also 842 kilometers longer than the coastline of North Carolina. Find the lengths of the coastlines of South Carolina and North Carolina.

13. **MUSIC DOWNLOADS** Denzel is comparing Web sites for downloading music. One charges a $5 membership fee plus $0.50 per track. Another charges $1.00 per track, but has no monthly fee. How many songs would Denzel have to buy for him to spend the same amount at both Web sites?

Solve each equation. Check your solution.

14. $3 + 2c = 2c$
15. $13y - 18 = -5y + 36$
16. $7d - 13 = 3d + 7$
17. $2f - 6 = 7f + 24$
18. $12n - 23.2 = -14n + 28.8$
19. $3.1w + 5 = 0.8 + w$

20. $10 - 2c = 2c$
21. $13y - 18 = -5y + 36$
22. $7d - 13 = 3d + 7$
23. $12n - 23.2 = -14n + 28.8$
24. $25.12 - x = 2x$
25. $3c + 4.5 = 7.2 - 6c$
26. $4.3n - 1.6 = 2.3n + 5.2$
27. $0.4x = 2x + 1.2$
28. $0.4x = 2x + 1.2$
29. $13y - 18 = -5y + 36$
30. $7d - 13 = 3d + 7$
31. $2f - 6 = 7f + 24$
32. $12n - 23.2 = -14n + 28.8$
33. **NUMBER SENSE** Three times the quantity $y + 7$ is equal to four times the quantity $y - 2$. What value of $y$ makes the sentence true?
34. OPEN ENDED Write an example of an equation with variables on each side. State the steps you would use to isolate the variable.

35. CHALLENGE An empty bucket is put under two faucets. If one faucet is turned on alone, the bucket fills in 6 minutes. If the other faucet is turned on alone, the bucket fills in 4 minutes. If both are turned on, how many seconds will it take to fill the bucket?

36. Writing in Math Explain how solving equations with variables on each side is like solving equations with variables on just one side. Include examples of both types of equations and an explanation of how they are alike and how they are different.

37. The formula \( F = \frac{9}{5} C + 32 \) is used to find the Fahrenheit temperature when a Celsius temperature is known. For what value are the Celsius and Fahrenheit temperatures the same?
   - A \( -72^\circ \)
   - B \( -40^\circ \)
   - C \( 0^\circ \)
   - D \( 32^\circ \)

38. Two weeks ago the sewing club had 1 less than 3 times their average attendance. Last week they had 3 more than their average attendance. If the attendance for both weeks were equal, what is the average attendance of the sewing club?
   - F 1
   - G 2
   - H 3
   - J 4

39. Olivia’s manager gave her a choice as to how she wants to be paid.

<table>
<thead>
<tr>
<th>Plan 1</th>
<th>Pay per Hour</th>
<th>Pay for Each Dollar of Appliance Sales</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$3</td>
<td>15¢</td>
</tr>
<tr>
<td></td>
<td>$4</td>
<td>10¢</td>
</tr>
</tbody>
</table>

Which equation shows what Olivia’s sales need to be in one hour to earn the same amount under either plan?
   - A \( 3 + 0.15s = 4 + 0.10s \)
   - B \( 3s + 0.15 = 4s + 0.10 \)
   - C \( 3 + 0.10s = 4 + 0.15s \)
   - D \( 3(s + 0.15) = 4(s + 0.10) \)

40. Find the true statement. (Lesson 7-8)
   - A line of fit is close to most of the data points.
   - A line of fit describes the exact coordinates of each point in the data set.
   - A line of fit always has a positive slope.

41. What equation represents the table of values? (Lesson 7-7)

<table>
<thead>
<tr>
<th>x</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>12</td>
</tr>
</tbody>
</table>

42. \( 4(x - 8) \)
43. \( 2(1.2c + 14) \)
44. \( \frac{1}{2}(n - 9) \)
Main Ideas
- Solve equations that involve grouping symbols.
- Identify equations that have no solution or an infinite number of solutions.

New Vocabulary
null or empty set
identity

Josh starts walking toward the park at a rate of 2 mph. One hour later, his sister Maria starts on the same path, riding her bike at 10 mph.

The table shows expressions for the distance Maria and Josh have traveled after a given time.

a. What does \( t \) represent?

b. Why is Maria’s time shown as \( t - 1 \)?

c. Write an equation that represents the time when Maria catches up to Josh. (Hint: They will have traveled the same distance.)

Solve Equations with Grouping Symbols
To find how many hours it takes Maria to catch up to Josh, you can solve the equation \( 2t = 10(t - 1) \). First, use the Distributive Property to remove the grouping symbols.

EXAMPLE

a. Solve the equation \( 2t = 10(t - 1) \). Check your solution.

\[
\begin{align*}
2t &= 10(t - 1) & & \text{Write the equation.} \\
2t &= 10t - 10(1) & & \text{Use the Distributive Property.} \\
2t &= 10t - 10 & & \text{Simplify.} \\
2t - 10t &= 10t - 10t - 10 & & \text{Subtract 10t from each side.} \\
-8t &= -10 & & \text{Simplify.} \\
\frac{-8t}{-8} &= \frac{-10}{-8} & & \text{Divide each side by -8.} \\
t &= \frac{5}{4} \text{ or } 1 \frac{1}{4} & & \text{Simplify.}
\end{align*}
\]

CHECK
Use dimensional analysis.

Josh traveled \( \frac{2 \text{ miles}}{\text{hour}} \times \frac{5 \text{ hour}}{4} \) or \( 2 \frac{1}{2} \) miles.

Maria traveled one hour less than Josh. She traveled \( \frac{10 \text{ miles}}{\text{hour}} \times \frac{1 \text{ hour}}{4} \) or \( 2 \frac{1}{2} \) miles.

Therefore, Maria caught up to Josh in \( \frac{1}{4} \) hour, or 15 minutes.
b. Solve \(5(a - 4) = 3(a + 1.5)\).

\[
\begin{align*}
5(a - 4) &= 3(a + 1.5) \\
5a - 20 &= 3a + 4.5 \\
5a - 20 + 20 &= 3a + 4.5 + 20 \\
5a &= 3a + 24.5 \\
5a - 3a &= 3a - 3a + 24.5 \\
2a &= 24.5 \\
\frac{2a}{2} &= \frac{24.5}{2} \\
a &= 12.25
\end{align*}
\]

Write the equation.
Use the Distributive Property.
Add 20 to each side.
Simplify.
Subtract 3a from each side.
Simplify.
Divide each side by 2.

**CHECK Your Progress**

Solve each equation. Check your solution.

1A. \(3x = 4(x + 2)\)

1B. \(-0.2(3c + 15) = 3(0.8c - 8)\)

Sometimes a geometric figure is described in terms of only one of its dimensions. To find the dimensions, you may have to solve an equation that contains grouping symbols.

**EXAMPLE**

**Use an Equation to Solve a Problem**

**GEOMETRY** The perimeter of a rectangle is 46 inches. Find the dimensions if the length is 5 inches greater than twice the width.

**Words**

2 times width + 2 times length = perimeter

**Variable**

Let \(w\) = the width.
Let \(2w + 5\) = the length.

**Equation**

\[
2w + 2(2w + 5) = 46
\]

Write the equation.
Use the Distributive Property.
Simplify.
Subtract 10 from each side.
Simplify.
Mentally divide each side by 6.

Evaluate \(2w + 5\) to find the length.

\(2(6) + 5 = 12 + 5 = 17\)

Replace \(w\) with 6.

The width is 6 inches. The length is 17 inches.

**2. RECYCLING** Sofia recycled 3 pounds less than 3 times the amount that James recycled. If they recycled a total of 53 pounds, how many pounds did each person recycle?

**Review Vocabulary**

**Perimeter** The distance around a geometric figure; Example: The perimeter of a square with sides that are 5 inches long is 20 inches. (Lesson 3-8)
**No Solution or All Numbers as Solutions** Some equations have no solution. That is, no value of the variable results in a true sentence. When this occurs, the set of solutions for the equation contains no elements. A set that contains no elements is called the null or empty set, shown by the symbol \( \emptyset \) or {}.

**EXAMPLE** No Solution

3
Solve \( 3x + \frac{1}{3} = 3x - \frac{1}{2} \).

\[
3x + \frac{1}{3} = 3x - \frac{1}{2} \\
\]
Write the equation.

\[
3x - 3x + \frac{1}{3} = 3x - 3x - \frac{1}{2} \\
\]
Subtract 3x from each side.

\[
\frac{1}{3} = -\frac{1}{2} \\
\]
Simplify.

The sentence \( \frac{1}{3} = -\frac{1}{2} \) is never true. So, the solution set is \( \emptyset \).

**Check Your Progress**

3. Solve \( 6x + 4 = 2(3x - 5) \). Check your solution.

An equation that is true for every value of the variable is called an identity.

**EXAMPLE** All Numbers as Solutions

4
Solve \( 2(2x - 1) + 6 = 4x + 4 \).

\[
2(2x - 1) + 6 = 4x + 4 \\
\]
Write the equation.

\[
4x - 2 + 6 = 4x + 4 \\
\]
Use the Distributive Property.

\[
4x + 4 = 4x + 4 \\
\]
Simplify.

\[
4x + 4 - 4 = 4x + 4 - 4 \\
\]
Subtract 4 from each side.

\[
x = x \\
\]
Simplify.

\[
\]
Mentally divide each side by 4.

The sentence \( x = x \) is always true. The solution set is all numbers.

**Check Your Progress**

4. Solve \( 20f + (-8f - 15) = 3(4f - 5) \). Check your solution.

**Example 1** (pp. 424–425)

Solve each equation. Check your solution.

1. \( 3(g - 3) = 6 \)  
2. \( 4(x + 1) = 28 \)  
3. \( 2(a - 2) = 3(a - 5) \)  
4. \( 16(z + 3) = 4(z + 9) \)  
5. \( 5(2c + 7) = 80 \)  
6. \( 6(3d + 5) = 75 \)  

**Example 2** (p. 425)

7. **GEOMETRY** The perimeter of a rectangle is 20 feet. The width is 4 feet less than the length. Find the dimensions of the rectangle.

**Examples 3, 4** (p. 426)

Solve each equation. Check your solution.

8. \( 12 - h = -h + 3 \)  
9. \( 3n + 4 = 3(n + 2) \)  
10. \( 3(2g + 4) = 6(g + 2) \)  
11. \( 4(f + 3) + 5 = 17 + 4f \)
Solve each equation. Check your solution.

12. \(2(d + 6) = 3d - 1\)  
13. \(6n - 18 = 4(n + 2.1)\)

14. \(3(a - 3) = 2(a + 4)\)  
15. \(3(s + 22) = 4(s + 12)\)

16. \(4(x - 2) = 3(1.5 + x)\)  
17. \(3(a - 1) = 4(a - 1.5)\)

18. \(2(3.5n + 6) = 2.5n - 2\)  
19. \(4.2x - 9 = 3(1.2x + 4)\)

20. \(2(3.5n + 6) = 2.5n - 2\)  
21. \(3(a - 1) = 4(a - 1.5)\)

22. \(8y - 5 = 5(y + 1) + 3y\)  
23. \(10z + 4 = 2(5z + 8) - 12\)

24. **GEOMETRY** The perimeter of a rectangle is 32 feet. Find the dimensions of the rectangle if the length is 4 feet longer than three times the width. Then find the area of the rectangle.

25. **BASKETBALL** Camilla has three times as many points as Lynn. Lynn has five more points than Kim. Camilla, Lynn, and Kim combined have twice as many points as Jasmine. If Jasmine has 25 points, how many points does each of the other three girls have?

Find the dimensions of each rectangle. The perimeter is given.

26. \(P = 460\) ft  
27. \(P = 440\) yd  
28. \(P = 11\) m

Solve each equation. Check your solution.

29. \(\frac{1}{2}(2n - 5) = 4n - 1\)  
30. \(y - 2 = \frac{1}{3}(y + 6)\)

31. \(-3(4b - 10) = \frac{1}{2}(24b + 60)\)  
32. \(\frac{3}{4}a + 4 = \frac{1}{4}(3a + 16)\)

33. \(0.4d = 2d + 1.24\)  
34. \(\frac{a - 6}{12} = \frac{a - 2}{4}\)

35. **GEOMETRY** The triangle and the rectangle have the same perimeter. Find the dimensions of each figure. Then find the perimeter of each figure.

36. **DECORATING** A gallon of paint covers about 350 square feet. A painter estimates the area to paint by multiplying the combined wall lengths by the height and subtracting 15 square feet for each window or door. Suppose a rectangular room measures 15 feet long by 12 feet wide. The room is 9 feet high and has two windows and two doors. How many gallons of paint are needed to paint the room using two coats of paint?

37. **OPEN ENDED** Give an example of an equation that has no solution and an equation that is an identity.

38. **CHALLENGE** An apple costs the same as 2 oranges. Together, an orange and a banana cost 10¢ more than an apple. Two oranges cost 15¢ more than a banana. What is the cost for one of each fruit?
39. **SELECT A TOOL/TECHNIQUE** Jamie has two spools with an equal length of plastic fencing that she is going to use to fence a rectangular and a triangular section of grass. The length of the rectangle will be 40 feet greater than the width, and the length of each side of the triangle will be 45 feet longer than the width of the rectangle. What technique(s) could be used to find the lengths of the sides? Justify your response and use your technique(s) to solve the problem.

- draw a model
- use a calculator
- use paper/pencil

40. **NUMBER SENSE** Three times the sum of three consecutive integers $x$, $x + 1$, and $x + 2$, is 72. What are the integers?

41. **Writing in Math** Why is the Distributive Property important for solving equations? Include in your answer a definition of the Distributive Property and a description of its use in solving equations.

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42. Wes leaves downtown driving 55 miles per hour. Emma follows 1 hour later, driving 60 miles per hour. Which equation can be used to determine how long it is after Wes leaves that Emma will catch up?

- **A** $55x = 60x - 1$
- **B** $60x = 55(x - 1)$
- **C** $55x = 60x$
- **D** $55x = 60(x - 1)$

43. Find the value of $x$ so that the polygons have the same perimeter.

- **F** 3
- **G** 6
- **H** 8
- **J** 12

---

**ALGEBRA** Solve each equation. Check your solution. (Lesson 8-1)

44. $4x = 2x + 5$

45. $3x + 5 = 7 - 2x$

46. $1.5x + 9 = 3x - 3$

**47. HOUSING** The table shows the median price of existing homes. Make a scatter plot and draw a line of fit for the data. Use the line of fit to predict the median price for an existing home in 2010. (Lesson 7-8)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Median Price (thousands)</td>
<td>97.1</td>
<td>110.5</td>
<td>128.4</td>
<td>139.0</td>
<td>158.1</td>
<td>170.0</td>
</tr>
</tbody>
</table>

Source: National Association of REALTORS

Express each number in scientific notation. (Lesson 4-7)

48. $4,500,000$

49. $-37,000$

50. $0.000498$

51. $-0.00203$

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**PREREQUISITE SKILL** Evaluate each expression. (Lesson 1-3)

52. $2t + 8, t = -3$

53. $b + 11, b = -15$

54. $4a, a = -6$
Meanings of at Most and at Least

The phrases *at most* and *at least* are used in mathematics. In order to use them correctly, you need to understand their meanings.

<table>
<thead>
<tr>
<th>Phrase</th>
<th>Meaning</th>
<th>Mathematical Symbol</th>
</tr>
</thead>
</table>
| at most  | • no more than
          | ≤                   |
|          | • less than or equal to  |                     |
| at least | • no less than
          | ≥                   |
|          | • greater than or equal to |                 |

Here is an example of one common use of each phrase, its meaning, and a mathematical expression for the situation.

**Verbal Expression**

You can spend *at most* $20.

**Meaning**

You can spend $20 or any amount less than $20.

**Mathematical Expression**

\[ s \leq 20, \text{ where } s \text{ represents the amount you spend.} \]

**Verbal Expression**

A person must be *at least* 18 to vote.

**Meaning**

A person who is 18 years old or any age older than 18 may vote.

**Mathematical Expression**

\[ a \geq 18, \text{ where } a \text{ represents age.} \]

Notice that the word *or* is part of the meaning in each case.

**Reading to Learn**

1. Write your own rule for remembering the meanings of *at most* and *at least*.

For each expression, write the meaning. Then write a mathematical expression using ≤ or ≥.

2. You need to earn at least $50 to help pay for a class trip.
3. The sum of two numbers is at most 6.
4. You want to drive at least 250 miles each day.
5. You want to hike 4 hours each day at most.
6. There are no more than 25 apples in the basket.
7. It will take at least 5 hours to finish this project.
Main Ideas
- Write inequalities.
- Graph inequalities.

New Vocabulary
inequality

Write Inequalities
A mathematical sentence that contains <, >, ≤, or ≥ is called an inequality.

EXAMPLE
Write Inequalities

Write an inequality for each sentence.

a. Your age is less than 6 years.

Words: Your age is less than 6 years.
Variable: Let \( a \) represent your age.
Inequality: \( a < 6 \)

b. Your speed is greater than 35 miles per hour.

Words: Your speed is greater than 35 miles per hour.
Variable: Let \( s \) represent your speed.
Inequality: \( s > 35 \)

CHECK Your Progress

1A. Your height is greater than or equal to 40 inches.
1B. Your speed is less than or equal to 35 miles per hour.
The table below shows some common verbal phrases and the corresponding mathematical inequalities.

<table>
<thead>
<tr>
<th>Inequalities</th>
<th>Inequalities</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; is less than</td>
<td>≥ is greater than or equal to</td>
</tr>
<tr>
<td>≤ is less than or equal to</td>
<td>≥ is greater than or equal to</td>
</tr>
<tr>
<td>&gt; is greater than</td>
<td>≤ is less than or equal to</td>
</tr>
<tr>
<td>≥ is greater than or equal to</td>
<td>≤ is less than or equal to</td>
</tr>
</tbody>
</table>

### Example: Determine Truth of an Inequality

**For the given value, state whether each inequality is true or false.**

**a.** \( s - 7 < 5, s = 14 \)

\[
\begin{align*}
14 - 7 &< 5 \\
7 &< 5
\end{align*}
\]

This sentence is false.

**b.** \( 12 \geq \frac{a}{2} + 2, a = 20 \)

\[
\begin{align*}
12 &\geq \frac{20}{2} + 2 \\
12 &\geq 12
\end{align*}
\]

Although the inequality \( 12 > 12 \) is false, the equation \( 12 = 12 \) is true. Therefore, this sentence is true.

### Check Your Progress

**3A.** \( 3 + x \leq 12, x = 6 \)

**3B.** \( y - 7 < 10, y = 17 \)
**Graph Inequalities** Inequalities can be graphed on a number line. The graph helps you visualize the values that make the inequality true.

### EXAMPLE

**Graph Inequalities**

Graph each inequality on a number line.

a. \( x > 4 \)

b. \( x \geq 4 \)

c. \( x < 4 \)

d. \( x \leq 4 \)

### CHECK Your Progress

4A. \( x < 5 \)  
4B. \( x \geq -2 \)  
4C. \( x > 0 \)  
4D. \( x \leq 2 \)

### EXAMPLE

**Write an Inequality**

Write the inequality for the graph.

An open circle is on 10, so the point 10 is *not* included in the graph. The arrow points to the right, so the graph includes all numbers greater than 10. The inequality is \( x > 10 \).

### CHECK Your Progress

5A. \( x < 5 \)  
5B. \( x \geq -2 \)

### Example 1

(p. 430)

Write an inequality for each sentence.

1. Lacrosse practice will be no more than 45 minutes.
2. Mario is more than 60 inches tall.

### Example 2

(p. 431)

3. **SOCCER** More than 8000 fans attended the Wizards’ opening soccer game at Arrowhead Stadium in Kansas City, Missouri. Write an inequality to describe the attendance.

### Example 3

(p. 431)

For the given value, state whether the inequality is *true* or *false*.

4. \( n + 4 > 6, n = 12 \)  
5. \( 34 \leq 4r, r = 8 \)
Write an inequality for each sentence.

12. The elevators in an office building have been approved for a maximum load of 3600 pounds.
13. Kyle’s earnings were no more than $60.
14. The race time of 86 minutes was greater than the winner’s time.
15. After a withdrawal, a savings account is now less than $500.

ANALYZE TABLES For Exercises 16 and 17 use the table that shows the average amount of time students ages 14 to 18 spend on homework per week.

<table>
<thead>
<tr>
<th>Group</th>
<th>Average Hours per week</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>5.4</td>
</tr>
<tr>
<td>Female</td>
<td>6.8</td>
</tr>
</tbody>
</table>

Source: Horatio Alger Association

16. Inali spends at least an hour more than the average time spent by boys on homework each week. Write an inequality for Inali’s homework time.
17. Anna usually spends no more than the average time spent by girls on homework each week. Write an inequality to represent Anna’s homework time.

For the given value, state whether each inequality is true or false.

18. \(18 - x > 4, x = 12\)
19. \(14 + n < 23, n = 8\)
20. \(5k > 35, k = 7\)
21. \(\frac{14}{c} < 7, c = 2\)
22. \(\frac{y}{3} \geq 2, y = 9\)
23. \(16 \leq 3d, d = 8\)

Graph each inequality on a number line.

24. \(a > 4\)
25. \(x > 6\)
26. \(d \leq 5\)
27. \(w \leq 8\)
28. \(n < 11\)
29. \(x < 5\)
30. \(t \geq 9\)
31. \(b \geq 8\)
32. \(x > -4\)
33. \(n \geq -3\)
34. \(x \leq -5\)
35. \(x < -2\)

Write the inequality for each graph.

36. \([-10, -9, -8, -7, -6, -5, -4, -3, -2]\)
37. \([-7, -6, -5, -4, -3, -2, -1, 0, 1]\)
38. \([-2, -1, 0, 1, 2, 3, 4, 5, 6]\)
39. \([7, 8, 9, 10, 11, 12, 13, 14, 15]\)

40. SPORTS There are more than 32,150 high school girls basketball and track programs in the United States. If there are 15,089 girls track programs, write and solve an inequality to determine the number of girls basketball programs.

41. RESEARCH Use the Internet or another source to find the state or national spending limits on certain government branches, organizations, or projects. Write an inequality to express one or more of these limits.
42. **NUMBER SENSE** Provide a counterexample to the statement, “All numbers less than 0 are negative integers.”

43. **OPEN ENDED** Write four examples of inequalities, each using one of the symbols <, >, ≤, and ≥. Tell the meaning of each inequality.

44. **CHALLENGE** Graph the solutions for each compound inequality.
   a. \( y < -2 \) or \( y > 3 \) (Hint: In a sentence, or means either part is true.)
   b. \( y \geq 0 \) and \( y \leq 5 \) (Hint: In a sentence, and means both parts must be true.)

45. **Writing in Math** How can inequalities help you describe relationships? Illustrate your answer with a real-world example that uses an inequality symbol and an explanation of the relationships described by the inequality.

---

**H.O.T. Problems**

46. Young adults are not allowed to vote in elections before their 18th birthday. Which graph represents the age of people who are allowed to vote?

   - **A**
   - **B**
   - **C**
   - **D**

47. Which inequality represents the graph below?

   - **F** \( x \geq -8 \)
   - **G** \( x \leq -8 \)
   - **H** \( x > -8 \)
   - **J** \( x < -8 \)

---

**ALGEBRA** Solve each equation. Check your solution. (Lesson 8-2)

48. \( 2(3 + x) = 14 \)
49. \( 63 = 9(2y - 3) \)
50. \( 3(n - 1) = 1.5(n + 2) \)

51. **ALGEBRA** Four times a number minus 6 is equal to the sum of 3 times the number and 2. Define a variable and write an equation to find the number. (Lesson 8-1)

52. \( \frac{2}{3} \div \frac{1}{3} \)
53. \( \frac{3}{2} \div \frac{1}{5} \)
54. \( \frac{1}{4} \div \frac{7}{2} \)

55. **MOWING** Keri has \( \frac{7}{8} \) gallon of gasoline left. Her mower uses \( \frac{1}{6} \) gallon to cut an average yard. How many average yards can she mow? (Lesson 5-4)

---

**PREREQUISITE SKILL** Solve each equation. (Lesson 3-3)

56. \( x + 19 = 32 \)
57. \( a + 7 = -3 \)
58. \( 26 + c = 19 \)
59. \( 44 - c = 26 \)
60. \( y - 9.7 = 10.1 \)
61. \( r - 1.6 = -0.6 \)
Main Idea

- Solve inequalities by using the Addition and Subtraction Properties of Inequality.

The paper bag on the balance may contain some blocks. The scale models an inequality because the two sides are not equal.

The side with the bag and 2 blocks weighs less than the side with 5 blocks. So, the inequality is $x + 2 < 5$.

a. How many blocks would be in the bag if the left side balanced the right side? (Assume that the paper bag weighs nothing.)

b. What numbers of blocks can be in the bag to make the left side weigh less than the right side?

c. Write an inequality to represent your answer to part b.

Solve Inequalities by Adding or Subtracting Solving an inequality means finding values for the variable that make the inequality true. In the example above, any number less than 3 is a solution. The solution is written as the inequality $x < 3$. You can solve inequalities by using the Addition and Subtraction Properties of Inequalities.

### EXAMPLE Solve an Inequality Using Subtraction

Solve $x + 3 > 10$. Check your solution.

1. \[ x + 3 > 10 \] Write the inequality.
2. \[ x + 3 - 3 > 10 - 3 \] Subtract 3 from each side.
3. \[ x > 7 \] Simplify.
To check your solution, try any number greater than 7.

**CHECK**  
\[ x + 3 > 10 \]  
Write the inequality.  
\[ 8 + 3 \nless 10 \]  
Replace \( x \) with 8.  
\[ 11 > 10 \checkmark \]  
This statement is true.

Any number greater than 7 will make the statement true. Therefore, the solution is \( x > 7 \).

1. Solve \( z + 4 > 3 \). Check your solution.

**EXAMPLE**  
Solve \(-6 \geq n - 5\). Check your solution.

\[-6 \geq n - 5\]  
Write the inequality.  
\[-6 + 5 \geq n - 5 + 5\]  
Add 5 to each side.  
\[-1 \geq n\]  
Simplify.  
The solution is \(-1 \geq n\) or \( n \leq -1\).

**CHECK Your Progress**

2. Solve \(-3 \geq g - 7\). Check your solution.

**EXAMPLE**  
Graph Solutions of Inequalities

3. Solve \( a + \frac{1}{2} < 2\). Graph the solution on a number line.

\[ a + \frac{1}{2} < 2 \]  
Write the inequality.  
\[ a + \frac{1}{2} - \frac{1}{2} < 2 - \frac{1}{2} \]  
Subtract \( \frac{1}{2} \) from each side.  
\[ a < \frac{4}{2} - \frac{1}{2} \]  
Rename 2 as a fraction with a denominator of 2.  
\[ a < \frac{3}{2} \text{ or } 1\frac{1}{2} \]  
Simplify.  
The solution is \( a < 1\frac{1}{2} \). Check your solution.

Graph the solution.

**CHECK Your Progress**

3. Solve \( b - 4 \geq 2\). Graph the solution on a number line.
STATE FAIRS  Antonio has at most $18 to ride go-carts and play games at the State Fair. If the go-carts cost $5.50, how much can he spend on games?

**Explore** We need to find the amount of money Antonio can spend on games.

**Plan** Let $x$ represent the amount Antonio can spend on games. Write an inequality to represent the problem. Recall that *at most* means *less than or equal to*.

<table>
<thead>
<tr>
<th>Words</th>
<th>Variable</th>
<th>Inequality</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost of go cart plus cost of games must be less than or equal to total amount</td>
<td>Let $x$ equal the cost of games.</td>
<td>$5.5 + x \leq 18$</td>
</tr>
</tbody>
</table>

**Solve**

\[
5.5 + x \leq 18 \\
5.5 - 5.5 + x \leq 18 - 5.5 \\
x \leq 12.5
\]

**Check** Check by choosing an amount less than or equal to $12.50, say, $10. Then Antonio would spend $5.50 + $10 or $15.50 in all. Since $15.50 < $18, the answer is reasonable.

So, at most, Antonio can spend $12.50 on games.

4. **HOMEWORK** Roman has at most three hours to work on a math assignment and a history project. If the math assignment will take $\frac{3}{4}$ hour, how much time can Roman spend working on his history project?

Personal Tutor at pre-alg.com

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### Examples 1, 2 (pp. 435–436)

Solve each inequality. Check your solution.

1. $x + 3 < 8$
2. $14 + y \geq 7$
3. $-13 \geq 9 + b$
4. $a - 5 > 6$
5. $c - (-2) \leq 3$
6. $-5 < t - 2$

### Example 3 (p. 436)

Solve each inequality. Then graph the solution on a number line.

7. $h + 4 > 4$
8. $x - 6 \leq 4$
9. $x + 3.75 \leq 5$
10. $7 > z + \frac{2}{3}$

### Example 4 (p. 437)

11. **SAVINGS** Chris is saving money to buy a stereo. He has $62.50, but his goal is to save at least $100. What is the least amount Chris still needs to save to reach his goal?
Solve each inequality. Check your answer.

12. \( p + 7 < 9 \)  
13. \( t + 6 > -3 \)  
14. \( -13 \geq 9 + b \)  
15. \( 16 > -11 + k \)  
16. \( 3 \geq -2 + y \)  
17. \( 25 < n + (-12) \)  
18. \( r - 5 \leq 2 \)  
19. \( a - 6 < 13 \)  
20. \( j - 8 \leq -12 \)  
21. \( -8 > h - 1 \)  
22. \( 22 > w - (-16) \)  
23. \( -30 \leq d + (-5) \)  

Solve each inequality. Then graph the solution on a number line.

24. \( n + 4 < 9 \)  
25. \( t + 7 > 12 \)  
26. \( p + (-5) > -3 \)  
27. \( -3 + z > 2 \)  
28. \( -13 \geq x - 8 \)  
29. \( -32 \geq a + (-5) \)  
30. \( 3 \leq \frac{1}{2} + a \)  
31. \( 4 \geq s - \frac{2}{3} \)  
32. \( -\frac{3}{4} < w - 1 \)  

33. **TRANSPORTATION** A certain minivan has a maximum carrying capacity of 1100 pounds. If the luggage weighs 120 pounds, what is the maximum weight allowable for passengers?

34. **MARINE BIOLOGY** Manatees can weigh up to 1000 pounds and are generally no more than 10 feet long. Suppose a manatee is currently 6.25 feet long. Write and solve an inequality to find how much longer the manatee could grow.

**HURRICANES** For Exercises 35–37, use the diagram below.

```
<table>
<thead>
<tr>
<th>Wind Speed of Storm (mph)</th>
<th>Depression</th>
<th>Tropical Storm</th>
<th>Hurricane</th>
</tr>
</thead>
<tbody>
<tr>
<td>39</td>
<td>74</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
```

35. A hurricane has winds that are at least 74 miles per hour. Suppose a tropical storm has winds that are 42 miles per hour. Write and solve an inequality to find how much the winds must increase so the storm is a hurricane.

36. Tropical storm Alpha has winds of 50 miles per hour. Write and solve an inequality to find how much the winds need to decrease so that the storm is downgraded to a depression.

37. A major storm has wind speeds that are at least 110 miles per hour. Write and solve an inequality that describes how much greater these wind speeds are than a hurricane with the slowest winds.

Solve each inequality. Check your solution.

38. \( 1 + y \leq 2.4 \)  
39. \( 2.9 < c + 7 \)  
40. \( f - 4 \geq 1.4 \)  
41. \( z - 2 > -3.8 \)  
42. \( b - \frac{3}{4} < \frac{2}{2} \)  
43. \( g - 1\frac{2}{3} > \frac{2}{6} \)  

44. **FIND THE DATA** Refer to the United States Data File on pages 18–21. Choose some data and write a real-world problem in which you would need to solve an inequality using addition or subtraction.
45. **OPEN ENDED** Write an inequality for the solution graphed below.

![Graph with x-axis from 16 to 24]

46. **FIND THE ERROR** Dylan and Jada are using the statement \( x \) minus three is greater than or equal to 15 to find values of \( x \). Who is correct? Explain.

Dylan
\[
\begin{align*}
    x - 3 &\geq 15 \\
    x - 3 + 3 &\geq 15 + 3 \\
    x &\geq 18
\end{align*}
\]

Jada
\[
\begin{align*}
    x - 3 &= 15 \\
    x - 3 + 3 &= 15 + 3 \\
    x &= 18
\end{align*}
\]

47. **CHALLENGE** Is it always, sometimes, or never true that \( x - 1 < x \)? Explain.

48. **Writing in Math** How is solving an inequality similar to solving an equation?

49. Trevor has $25 to spend on a T-shirt and shorts for gym class. The shorts cost $14. Based on the inequality \( 14 + t \leq 25 \), where \( t \) represents the cost of the T-shirt, what is the most Trevor can spend on the T-shirt?
   
   A $9  
   B $10.99  
   C $11  
   D $11.50

50. The length of the rectangle is greater than its width. Which inequality represents the possible values of \( x \)?

   A \( x \leq 17 \)  
   B \( x \geq 17 \)  
   C \( x < 17 \)  
   D \( x > 17 \)

ALGEBRA For the given value, state whether each inequality is true or false. (Lesson 8-3)

51. \( x - 5 > 4, x = 9 \)

52. \( 9 + a \leq 3, a = -7 \)

53. \( d \geq 8, d = 4 \)

54. **GEOMETRY** The perimeter of a rectangle is 24 centimeters. Find the dimensions if the length is 3 more than twice the width. (Lesson 8-2)

55. \( 4(2 + 8) \)

56. \( -2(n + 6) \)

57. \( 5(x - 3.5) \)

58. \( (9 - d)(-3c) \)

ALGEBRA Use the Distributive Property to rewrite each expression as an equivalent algebraic expression. (Lesson 3-1)

59. \( -7x = 14 \)

60. \( -3y = -27 \)

61. \( \frac{d}{-3} = -6 \)

62. \( \frac{c}{-4} = 12 \)

**GET READY for the Next Lesson**

**PREREQUISITE SKILL** Solve each equation. (Lesson 3-4)
Solve each equation. Check your solution.  
(Lesson 8-1)
1. $6y + 42 = 4y$
2. $12x - 19 = 3x + 8$
3. $7m - 12 = 2.5m + 2$

Define a variable and write an equation to find each number. Then solve.  
(Lesson 8-1)
4. Twice a number is 150 less than 5 times the number. What is the number?
5. One fourth of a number plus 3 is $\frac{1}{2}$ that number minus 1. What is the number?

6. TESTS  Bobby’s score is 5 less than twice Allan’s score. It is also 45 points greater than Allan’s score. What score did the two boys receive?  
(Lesson 8-1)

7. MULTIPLE CHOICE  An online computer game community has two membership plans. The first plan gives you unlimited play time for $40 a month. The second plan charges a monthly access fee of $4.25 plus $2.75 for each hour you play. After how many hours do the two plans cost the same amount?  
(Lesson 8-1)
A 6.6  B 9.0  C 11.2  D 13.0

Solve each equation. Check your solution.  
(Lesson 8-2)
8. $8(p - 4) = 2(2p + 1)$
9. $0.2x - 1.4 = 15.82 - 0.5x$
10. $b + 2(b + 5) = 3(b - 1) + 13$

11. MULTIPLE CHOICE  Which of the following graphs represents the inequality $-\frac{17}{5} \leq y$?  
(Lesson 8-3)
F
G
H
J

Graph each inequality on a number line.  
(Lesson 8-3)
12. $x < -3$
13. $y \geq 5$
14. $\frac{4}{5} > d$
15. $f < 11.2$

Write an inequality for each sentence.  
(Lesson 8-3)
16. More than 35,000 people attended a concert in Toronto.
17. Toby wants to spend no more than 3 hours working on his model car.
18. FITNESS  The table shows a gym class’s average results for boys and girls participating in the long jump.

<table>
<thead>
<tr>
<th>Gender</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>17 feet 5 inches</td>
</tr>
<tr>
<td>Female</td>
<td>14 feet 3 inches</td>
</tr>
</tbody>
</table>

Cheyenne could jump no farther than 12 inches more than the average distance for males. Write an inequality that gives the possible distances that Cheyenne could jump.  
(Lesson 8-3)

Solve each inequality. Check your solution.  
(Lesson 8-4)
19. $d + 10 \geq 12$
20. $c - (-5) < 24$
21. $5 < g - 21$
22. $-32 \leq 17 + j$
23. $k - 3 > 7$
24. $7 \leq m + 1$

25. MULTIPLE CHOICE  Shanté has $50 to spend on a back-to-school outfit. The blouse she wants is $17. Based on the inequality $17 + s \leq 50$ where $s$ is the cost of a skirt, what is the most that Shanté can spend on a skirt?  
(Lesson 8-4)
A $17  
B $33  
C $43  
D $50
Solving Inequalities by Multiplying or Dividing

Main Ideas
- Solve inequalities by multiplying or dividing by a positive number.
- Solve inequalities by multiplying or dividing by a negative number.

**GET READY for the Lesson**

An astronaut in a space suit weighs about 300 pounds on Earth, but only 50 pounds on the Moon because of weaker gravity.

<table>
<thead>
<tr>
<th>Location</th>
<th>Weight of Astronaut (lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earth</td>
<td>300</td>
</tr>
<tr>
<td>Moon</td>
<td>50</td>
</tr>
<tr>
<td>Pluto</td>
<td>67</td>
</tr>
<tr>
<td>Mars</td>
<td>113</td>
</tr>
<tr>
<td>Neptune</td>
<td>407</td>
</tr>
<tr>
<td>Jupiter</td>
<td>796</td>
</tr>
</tbody>
</table>

If the astronaut and space suit each weighed half as much, would the inequality still be true? That is, would the astronaut’s weight still be greater on Earth?

a. Divide each side of the inequality $300 > 50$ by 2. Is the inequality still true? Explain by using an inequality.

b. Would the weight of 5 astronauts be greater on Pluto or on Earth? Explain by using an inequality.

**Multiply or Divide by a Positive Number** The application above demonstrates how you can solve inequalities by using the Multiplication and Division Properties of Inequalities.

**KEY CONCEPT**

<table>
<thead>
<tr>
<th>Words</th>
<th>Symbols</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>When you multiply or divide each side of an inequality by the same positive number, the inequality remains true.</td>
<td>For all numbers $a$, $b$, and $c$, where $c &gt; 0$, 1. if $a &gt; b$, then $ac &gt; bc$ and $\frac{a}{c} &gt; \frac{b}{c}$. 2. if $a &lt; b$, then $ac &lt; bc$ and $\frac{a}{c} &lt; \frac{b}{c}$.</td>
<td>$2 &lt; 6$   [\frac{3}{3} &gt; \frac{-9}{3}] $4 \cdot 2 &lt; 4 \cdot 6$ $\frac{3}{3} &gt; \frac{-9}{3}$ $8 &lt; 24$ $1 &gt; -3$</td>
</tr>
</tbody>
</table>

These properties are also true for $a \geq b$ and $a \leq b$. 

**Study Tip**

**Positive Number**
The inequality $c > 0$ means that $c$ is a positive number.
EXAMPLE Multiply or Divide by a Positive Number

Solve each inequality. Check your solution.

1. \(8x \leq 40\)
   a. \(8x \leq 40\) Write the inequality.
   \[
   \frac{8x}{8} \leq \frac{40}{8}
   \]
   \(x \leq 5\) Simplify.
   The solution is \(x \leq 5\). You can check this solution by substituting 5 or a number less than 5 into the inequality.

   b. \(\frac{d}{2} > 7\)
   \[
   \frac{d}{2} > 7
   \]
   \[2\left(\frac{d}{2}\right) > 2(7)\] Multiply each side by 2.
   \(d > 14\) Simplify.
   The solution is \(d > 14\). You can check this solution by substituting a number greater than 14 into the inequality.

1A. \(3x > -15\) 
1B. \(\frac{f}{4} < -5\)

CHECK Your Progress

STANDARDIZED TEST EXAMPLE

Ling earns $8 per hour. Which inequality can be used to find how many hours he must work in a week to earn at least $120?
A. \(8x < 120\)  
B. \(8x \leq 120\)  
C. \(8x > 120\)  
D. \(8x \geq 120\)

Read the Test Item

You are to write an inequality to represent a real-world problem.

Solve the Test Item

**Words**
- Amount earned per hour
- times
- number of hours
- is at least
- the amount earned each week

**Variable**
- Let \(x\) represent the number of hours worked.

**Inequality**
- \(8 \cdot x \geq 120\)

The answer is D.

2. It takes Alfonzo \(\frac{3}{4}\) hour to mow a lawn. Which inequality can be used to find the number of lawns he can mow if he works 15 hours per week?
F. \(\frac{3}{4}x \leq 15\)  
G. \(\frac{3}{4}x > 15\)  
H. \(\frac{3}{4}x > 15\)  
J. \(\frac{3}{4}x < 45\)

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Multiplication and Division Properties

Lesson 8-5
Solving Inequalities by Multiplying or Dividing

Negative Number
The inequality $c < 0$ means that $c$ is a negative number.

EXAMPLE
Multiply or Divide by a Negative Number
Solve each inequality and check your solution. Then graph the solution on a number line.

a. $x - 3 \leq 4$

- Write the inequality.
- $\frac{x}{-3} \leq 4$
- Multiply each side by $-3$ and reverse the symbol.
- $-3 \left( \frac{x}{-3} \right) \geq -3(4)$
- $x \geq -12$
- Check this result.
- Graph the solution, $x \geq -12$.

b. $-7x > -56$

- Write the inequality.
- $\frac{-7x}{-7} < \frac{-56}{-7}$
- Divide each side by $-7$ and reverse the symbol.
- $x < 8$
- Check this result.
- Graph the solution, $x < 8$.

Extra Examples at pre-alg.com

KEY CONCEPT

Multiplication and Division Properties

Words When you multiply or divide each side of an inequality by the same negative number, the inequality symbol must be reversed for the inequality to remain true.

Symbols For all numbers $a, b$, and $c$, where $c < 0$,
1. if $a > b$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$.
2. if $a < b$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$.

Examples

- $7 > 1$
- $-4 < 16$
- $-2(7) < -2(1)$ Reverse the symbols.
- $\frac{-4}{-4} > \frac{16}{-4}$
- $-14 < -2$
- $1 > -4$

These properties are also true for $a \geq b$ and $a \leq b$. 
Solve each inequality. Check your solution.

1. \(2x < 8\)
2. \(3x \geq -6\)
3. \(4x < 4\)
4. \(\frac{7}{2}y > 63\)
5. \(\frac{3}{4} \leq \frac{5}{7}y\)
6. \(\frac{3}{24}y \leq \frac{1}{4}\)

Example 2 (p. 442)

7. **MULTIPLE CHOICE** Koto delivers pizzas on weekends. Her average tip is $1.50 for each pizza that she delivers. How many pizzas must she deliver to earn at least $20 in tips?

A. 10  
B. 13  
C. 14  
D. 20

Example 3 (p. 443)

Solve each inequality. Check your solution. Then graph the solution on a number line.

8. \(-4t > -20\)
9. \(-8z \leq -24\)
10. \(18 > -\frac{2}{3}g\)

**HOMEWORK HELP**

<table>
<thead>
<tr>
<th>HOMEWORK HELP</th>
<th>For Exercises</th>
<th>See Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>11–18</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>19, 20, 42, 44</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>21–28</td>
<td>3</td>
</tr>
</tbody>
</table>

**Exercises**

Solve each inequality. Check your solution.

11. \(13a \geq -26\)
12. \(-15 \leq 5b\)
13. \(144 < 12d\)
14. \(15 \geq 3t\)
15. \(\frac{p}{6} > 5\)
16. \(7 \geq \frac{h}{14}\)
17. \(3m < 33\)
18. \(8z \leq -24\)

19. **SOCCER** Tomás wants to spend less than $100 for a new soccer ball and shoes. The ball costs $24. Write and solve an inequality that gives the amount that Tomás can spend on shoes.

20. **ARCADE** Montel spends $0.75 every time he plays his favorite video game. Montel has $10. Write and solve an inequality that shows how many times Montel can play the video game.

Solve each inequality. Check your solution. Then graph the solution on a number line.

21. \(-8 \leq -4w\)
22. \(-6a > -78\)
23. \(-25t \leq 400\)
24. \(18 > -2g\)
25. \(-\frac{y}{4} \geq 2.4\)
26. \(-\frac{n}{5} \geq -0.8\)
27. \(6 > \frac{x}{7}\)
28. \(-\frac{r}{2} < -2\)

29. **SWIMMING** Andrea swims 40 meters per minute, and she wants to swim at least 2000 meters this morning. Write and solve an inequality to find how long she should swim.

Solve each inequality. Check your solution. Then graph the solution on a number line.

30. \(-5 \geq -\frac{c}{4.5}\)
31. \(-19 > \frac{y}{-0.3}\)
32. \(-\frac{1}{3}x \geq -9\)
33. \(-36 < -\frac{1}{2}b\)
34. \(\frac{y}{-3} < -7\)
35. \(\frac{k}{-2} < 9\)
36. \(-8 > \frac{k}{-0.4}\)
37. \(\frac{m}{-7} \leq 1.2\)

38. **OPEN ENDED** Write an inequality that can be solved using the Division Property of Inequality, where the inequality symbol is not reversed.

39. **CHALLENGE** The product of an integer and \(-7\) is less than \(-84\). Find the least integer that meets this condition.
40. **FIND THE ERROR** Brittany and Tamika each solved \(-45 \geq 9k\). Who is correct? Explain your reasoning.

Brittany
\[-45 \geq 9k\]
\[-\frac{45}{9} \geq \frac{9k}{9}\]
\[-5 \leq k\]

Tamika
\[-45 \geq 9k\]
\[-\frac{45}{9} \geq \frac{9k}{9}\]
\[-5 \geq k\]

41. **Writing in Math** Use the information on page 441 to explain how inequalities can be used in studying space. Illustrate your answer with inequalities that compare the weight of two astronauts on Mars and on the Moon.

42. **ALGEBRA** Solve each inequality. Check your solution. (Lessons 8-4)

45. \(-4 + x > 23\)

46. \(c + 18 \leq -2\)

47. \(6 > n - 10\)

48. **CRAFTS** It takes Carolyn two hours to complete a cross-stitch pattern. Carolyn can spend no more than fourteen hours cross-stitching. Write an inequality that represents this situation and use it to determine whether Carolyn can complete 8 cross-stitch patterns. (Lesson 8-3)

49. \(\frac{1}{8} \cdot \frac{3}{4}\)

50. \(-\frac{3}{7} \cdot \frac{5}{9}\)

51. \(2\frac{1}{2} \cdot \left(-\frac{5}{6}\right)\)

52. \(\frac{ab}{2} \cdot \frac{4}{bc}\)

53. \(2x + 3 = 9\)

54. \(5a - 6 = 14\)

55. \(3n - 8 = -26\)

56. \(\frac{t}{3} + 5 = 2\)

57. \(\frac{c}{4} - 1 = 4\)

58. \(\frac{d}{2} + 3 = 19\)
**Main Idea**

- Solve inequalities that involve more than one operation.

**Inequalities with More than One Operation** An inequality may involve more than one operation. To solve the inequality, work backward to undo the operations, just as you did in solving multi-step equations.

**EXAMPLE** Solve a Two-Step Inequality

1. Solve $6x + 15 > 9$ and check your solution. Graph the solution on a number line.

   - **Write the inequality.**
   - $6x + 15 > 9$
   - **Subtract 15 from each side.**
   - $6x > -6$
   - **Simplify.**
   - $x > -1$
   - **Mentally divide each side by 6.**

   **CHECK**
   - **Write the inequality.**
   - $6x + 15 > 9$
   - **Replace x with a number greater than $-1$. Try 0.**
   - $6(0) + 15 > 9$
   - **Simplify.**
   - $0 + 15 > 9$
   - $15 > 9\checkmark$
   - **The solution checks.**

   Graph the solution, $x > -1$.

**CHECK Your Progress**

1. Solve $8y - 2 \leq 14$ and check your solution. Graph the solution on a number line.
EXAMPLE
Reverse the Inequality Symbol

Solve $10 - 3a \leq 25 + 2a$ and check your solution. Graph the solution on a number line.

1. Write the inequality.
2. Subtract $2a$ from each side.
4. Subtract 10 from each side.
5. Simplify.
6. Divide each side by $-5$ and change $\leq$ to $\geq$.
7. Simplify.

Check your solution by substituting a number greater than $-3$. Graph the solution, $a \geq -3$.

When inequalities contain grouping symbols, you can use the Distributive Property to begin simplifying the inequality.

RUNNING Refer to the application at the beginning of the lesson.

Tammy wants to be able to run at least the standard marathon distance of 26.2 miles. If the length of her current daily runs is about 4 miles, how many miles should she increase her daily run by to meet her goal?

- **Words**: 3 times 4 miles plus amount of increase is greater than or equal to desired distance.
- **Variable**: Let $d = \text{the amount of increase}$.
- **Inequality**: $3 \cdot (4 + d) \geq 26.2$

Write the inequality. Multiply.
12 + $3d \geq 26.2$
Subtract 12 from each side.
$3d \geq 14.2$
Divide each side by 3.
$rac{3d}{3} \geq \frac{14.2}{3}$

In order to have enough endurance to run a marathon, Tammy should increase the distance of her average daily run by at least 4.73 miles.
3. **BUSINESS** Banks estimate the value of a business to determine loans and insurance. The formula for the value of a coffee shop is 40% of its annual sales plus the value of its inventory. The value of Holmes Coffee is at least $150,000. Write and solve an inequality to find the annual sales at Holmes Coffee if its inventory is $26,000.

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---

**Solve each inequality and check your solution. Then graph the solution on a number line.**

**Example 1** (p. 446)

1. \(3x + 4 \leq 31\)
2. \(12a - 4 > 20\)
3. \(2n + 5 > 11 - n\)
4. \(y + 1 \geq 4y + 4\)
5. \(16 - 2c < 14\)
6. \(18 \leq 12 - 2n\)
7. \(-3(b - 1) > 18\)
8. \(-2(k + 1) \geq 16\)

**Example 2** (p. 447)

9. **MONEY** A company pays Dante’s Web site for advertising on the site. The Web site earns $10 per month plus $0.05 each time a visitor to the site clicks on the advertisement. What is the least number of clicks he needs to make $45 per month or more from this advertiser?

**Example 3** (p. 447)

---

**Solve each inequality and check your solution. Then graph the solution on a number line.**

**Exercises**

**For Exercises 10–15, 16–19, 20–23, see Examples 1, 2, 3.**

10. \(2x + 8 > 24\)
11. \(6q + 4 \leq 28\)
12. \(3y - 1 \leq 5\)
13. \(9t - 5 \leq -14\)
14. \(3 + 4c > -13\)
15. \(9 + 2p \leq 15\)
16. \(4 - 3k \leq 19\)
17. \(16 - 4n > 20\)
18. \(-3b + 4 < -2\)
19. \(-5a - 8 > 12\)
20. \(2(n + 3) < -4\)
21. \(2(d + 1) > 16\)

**For Exercises 22 and 23, write and solve an inequality.**

22. **SALES** You earn $2 for every magazine subscription you sell plus a salary of $10 each week. How many subscriptions do you need to sell each week to earn at least $40 each week?

23. **HIKING** You hike along the Appalachian Trail at 3 miles per hour. You stop for one hour for lunch. You want to walk at least 18 miles. How many hours should you expect to spend on the trail?

**Solve each inequality and check your solution. Then graph the solution on a number line.**

24. \(3x - 2 > 10 - x\)
25. \(c - 1 < 3c + 5\)
26. \(2 + 0.3y \geq 11\)
27. \(0.5a - 1.4 \leq 2.1\)
28. \(\frac{1}{2}(6 - c) > 5\)
29. \(\frac{m}{2} + 9 \geq 5\)
30. Four times a number less 6 is greater than two times the same number plus 8. For what number or numbers is this true?
31. One half of the sum of a number and 6 is less than 25. For what numbers is this true?

32. **REAL ESTATE** A new real estate agent receives a monthly salary of $1500 plus a 3.5% commission on every home sold. For what amount of monthly sales will the agent earn at least $5000?

33. **REPAIRS** Carl is having a mechanic fix his car. The mechanic said that the job was going to cost at least $375 for parts and labor. If the cost of the parts was $150, and the mechanic charges $60 an hour, how many hours is the mechanic planning on working on the car?

34. **SCHOOL** Nate has scores of 85, 91, 89, and 93 on four tests. What is the least number of points he can get on the fifth test to have an average of at least 90?

35. **FUND-RAISERS** The booster club at Jefferson High School sells football programs for $1 each. The costs to make the programs are $60 for page layout plus $0.20 for printing each program. If they print 400 programs, how many programs must the Club sell to make at least $200 profit?

36. **CAR RENTAL** The costs for renting a car from Able Car Rental and from Baker Car Rental are shown in the table. For what mileage does Baker have the better deal? Use the inequality $30 + 0.05x > 20 + 0.10x$. Explain why this inequality works.

37. **CELL PHONE SERVICES** While reviewing prepay phone plans, Miko found that FoneCom charges a $5.35 monthly fee plus $0.10 per minute. Miko currently has BestPhone service at $10 per month plus $0.05 per minute. Miko figures that her monthly bill would be more with FoneCom. For how many minutes per month does she use the phone?

38. **TRAVEL** Tim is taking the train to Seattle to visit his grandparents. He was given $5.00 to spend on snacks and reading material. Granola bars cost $0.75 each and a newspaper is $1.25. If Tim buys a newspaper, how many granola bars can he get?

39. **OPEN ENDED** Write a multi-step inequality that can be solved by first adding 3 to each side.

40. **CHALLENGE** Assume that $k$ is an integer. Solve the inequality $10 - 2|k| > 4$.

41. **FIND THE ERROR** Jerome and Ryan are solving $2(2y + 3) > y + 1$. Who is correct? Explain your reasoning.

42. **Writing in Math** Use the information about running found on page 446 to explain how multi-step inequalities are used in running.
43. Sandra’s scores on the first five science tests are shown in the table. Which inequality represents the score she must receive on the sixth test to have an average score of more than 88?

<table>
<thead>
<tr>
<th>Test</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>85</td>
</tr>
<tr>
<td>2</td>
<td>84</td>
</tr>
<tr>
<td>3</td>
<td>90</td>
</tr>
<tr>
<td>4</td>
<td>95</td>
</tr>
<tr>
<td>5</td>
<td>88</td>
</tr>
</tbody>
</table>

A. \(s \geq 86\)
B. \(s \leq 88\)
C. \(s < 88\)
D. \(s > 86\)

44. An art teacher wants to buy at least 2 canvases for each student in her painting class. If there are 30 students in the class and if canvases cost $16 per package, what other information is needed to find the amount the teacher should budget for canvases?

F. The number of art projects planned for the course
G. The cost of paints and brushes
H. The number of canvases in each package
J. The total budget available for art supplies

45. \(6x < -27\)
46. \(-5m \geq -15\)
47. \(8 > \frac{q}{3}\)
48. \(\frac{n}{-4} \leq -11\)
49. \(-9 + k > 20\)
50. \(22 \leq -15 + y\)
51. \(12 + z \leq 8\)
52. \(14 \geq 7 + a\)

53. **SCHOOL** If 12 of the 20 students in a class are boys, what percent are boys? (Lesson 6-5)

54. Write \(\frac{1}{200}\) as a percent. (Lesson 6-5)

55. $5 for 2 loaves of bread
56. 200 miles on 12 gallons
57. 24 meters in 4 seconds
58. 9 monthly issues for $11.25

59. **GEOMETRY** Find the missing dimension in each rectangle. (Lesson 3-8)

\[
\begin{align*}
18.4 \text{ ft} & \quad \text{Perimeter} = 49.6 \text{ ft} \\
? & \\
\end{align*}
\]

60. \(\ell\)

\[
\begin{align*}
5.1 \text{ m} & \quad \text{Area} = 30.6 \text{ m}^2 \\
\end{align*}
\]

**Cross-Curricular Project**

**Math and Recreation**

**Just for Fun** It is time to complete your project. Use the information and data you have gathered about recreational activities to prepare a Web page or poster. Be sure to include a scatter plot and a prediction for each activity.

**Cross-Curricular Project at** [pre-alg.com]
Key Concepts

Solving Equations (Lessons 8-1 and 8-2)

- Use the Addition or Subtraction Property of Equality to isolate the variables on one side of an equation.
- Use the Distributive Property to remove the grouping symbols.

Solving Inequalities (Lessons 8-3 to 8-6)

- An inequality is a mathematical sentence that contains $<$, $>$, $\leq$, or $\geq$.
- Solving an inequality means finding values for the variable that make the inequality true.
- When you multiply or divide each side of an inequality by a positive number, the inequality symbol remains the same.
- When you multiply or divide each side of an inequality by a negative number, the inequality symbol must be reversed.
- To solve an inequality that involves more than one operation, work backward to undo the operations.

Vocabulary Check

Determine whether each statement is true or false. If false, replace the underlined word or phrase to make a true statement.

1. When an equation has no solution, the solution set is the null set.
2. The inequality $n + 8 - 8 \geq 14 - 8$ demonstrates the Subtraction Property of Inequality.
3. An equation that is true for every value of the variable is called an inequality.
4. The inequality $\frac{x}{4}(4) < 7(4)$ demonstrates the Division Property of Inequality.
5. A mathematical sentence that contains $<$, $>$, $\leq$, or $\geq$ is called an empty set.
6. When the final result in solving an equation is $5 = -8$, the solution set is the null set.
7. The symbol $\geq$ means is less than or equal to.
Lesson-by-Lesson Review

8–1 Solving Equations with Variables on Each Side

Solve each equation. Check your solution.

8. \(2a + 9 = 5a\)  
9. \(x - 4 = 3x\)  
10. \(3y - 8 = y\)  
11. \(19t = 26 + 6t\)  
12. \(12 + 1.5x = 9\)  
13. \(5b - 1 = 2.5b - 4\)

14. CONCERTS An outdoor concert venue is planning on increasing the number of concerts by \(\frac{1}{14}\) for next year. This will increase their number of concerts by 3. How many concerts will they host this year?

15. An online DVD rental club has two membership plans as shown. In how many months would the total cost of the two plans be the same?

<table>
<thead>
<tr>
<th>Plan</th>
<th>Membership Fee</th>
<th>Cost Per Month</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$20</td>
<td>$5</td>
</tr>
<tr>
<td>B</td>
<td>$30</td>
<td>$3</td>
</tr>
</tbody>
</table>

Example 1

Solve \(7x = 3x - 12\).

\[
7x = 3x - 12
\]

Write the equation.

\[
7x - 3x = 3x - 3x - 12
\]

Subtract \(3x\) from each side.

\[
x = -12
\]

Simplify.

\[
\frac{4x}{4} = \frac{-12}{4}
\]

Divide each side by 4.

\[
x = -3
\]

Simplify.

8–2 Solving Equations with Grouping Symbols

Solve each equation. Check your solution.

16. \(4(k + 1) = 16\)  
17. \(2(n - 5) = 8\)  
18. \(11 + 2q = 2(q + 4)\)  
19. \(\frac{1}{2}(t + 8) = \frac{3}{4}t\)  
20. \(4(x + 2.5) = 3(7 + x)\)  
21. \(3(x + 1) - 5 = 3x - 2\)

Example 2

Solve \(2(x + 3) = 15\).

\[
2(x + 3) = 15
\]

Write the equation.

\[
2x + 6 = 15
\]

Use the Distributive Property.

\[
x = 9
\]

Subtract 6 from each side.

\[
\frac{2x}{2} = \frac{9}{2}
\]

Simplify.

\[
x = 4.5
\]

Divide each side by 2.

22. GEOMETRY The perimeter of a rectangle is 84 meters. Find the dimensions of the rectangle if the length is 3 meters less than twice the width.
Inequalities (pp. 430–434)

For the given value, state whether each inequality is true or false.

23. \( x + 4 > 9, x = 12 \)
24. \( 12 - t < 5, t = 3 \)
25. \( 6r > 30, r = 5 \)
26. \( 15 \leq 5n, n = 8 \)
27. \( 3n + 1 \geq 14, n = 7 \)
28. \( 23 \leq \frac{c}{4} + 2, c = 10 \)

29. CAMPING When camping, Stephan and his friends usually use at least 3 logs for fire each night. Write an inequality that represents this situation.

30. DIVING In a diving competition, the diver in first place has a total score of 345.4. Ming has scored 68.2, 68.9, 67.5, and 71.7 for her first four dives and has one more dive remaining. Write an inequality to show the score \( x \) that Ming must receive on her fifth dive in order to overtake the diver in first place.

Example 3 State whether \( n + 11 < 14 \) is true or false for \( n = 5 \).

\[
\begin{align*}
  n + 11 &< 14  \\
  5 + 11 &< 14  \\
  16 &< 14
\end{align*}
\]

The sentence is false.

Solving Inequalities by Adding or Subtracting (pp. 435–439)

Solve each inequality. Then graph the solution on a number line.

31. \( b - 9 \geq 8 \)
32. \( 15 > 3 + n \)
33. \( x + 4.8 \leq 2 \)
34. \( r + 5.7 \leq 6.1 \)
35. \( t + \frac{1}{2} < 4 \)
36. \( -1\frac{2}{5} < k - 3 \)

37. MOVING A moving company is loading a 920-pound piano into a service elevator. The elevator can carry a maximum of 1800 pounds. Write and solve an inequality to determine how much additional weight the elevator can carry.

Example 4 Solve \( x - 7 \leq 3 \). Then graph the solution on a number line.

\[
\begin{align*}
  x - 7 &\leq 3  \\
  x - 7 + 7 &\leq 3 + 7  \\
  x &\leq 10
\end{align*}
\]

\[
\begin{align*}
  2 & 3 4 5 6 7 8 9 10 11 12
\end{align*}
\]
8–5 Solving Inequalities by Multiplying or Dividing  (pp. 441–445)

Solve each inequality. Then graph the solution on a number line.

38. \( \frac{n}{4} < 6 \)  
39. \( \frac{k}{1.7} \leq 3 \)  
40. \( 0.5x > 3.2 \)  
41. \( -56 \geq 8y \)  
42. \( 9 > \frac{x}{-4} \)  
43. \( \frac{-5}{6} a \leq 2 \)

44. **GOLF**  Gazelle wants to spend less than $38.50 on new golf balls. If each box costs $11, what is the maximum number of boxes of golf balls that she can buy?

45. **JOBS**  Dakota earns $8 per hour working at a landscaping company and wants to earn at least $1200 this summer.
   a. Write an inequality to represent this situation.
   b. Solve the inequality that you found in part a.
   c. What is the minimum number of hours Dakota will have to work?

---

8–6 Solving Multi-Step Inequalities  (pp. 446–450)

Solve each inequality. Check your solution.

46. \( 2x - 3 > 19 \)  
47. \( 5n + 4 \leq 24 \)  
48. \( 6 \geq \frac{r}{7} + 1 \)  
49. \( \frac{t}{-2} + 15 < 21 \)  
50. \( 3(a + 8.4) > 30 \)  
51. \( \frac{1}{4} + 2b < 13 + 5b \)

52. **SALES**  A car sales associate receives a monthly salary of $1700 a month plus 8% commission on every car sold. For what amount of monthly sales will the sales associate earn at least $4200?

---

**Example 5**  Solve \(-2n \geq 26\). Then graph the solution on a number line.

\[-2n \geq 26 \]
Write the inequality.

\[-2n \geq 26 \]
Divide each side by \(-2\) and reverse the symbol.

\[-2 \]
Simplify.

\[-13 \]

---

**Example 6**  Solve \(4t + 7 < -5\).

\[4t + 7 < -5\]
Write the inequality.

\[4t + 7 - 7 < -5 - 7\]
Subtract 7 from each side.

\[4t < -12\]
Simplify.

\[t < -3\]
Mentally divide each side by 4.
Solve each equation. Check your solution.

1. $7x - 3 = 10x$
2. $p - 9 = 4p$
3. $2(6 - 5d) = -8$
4. $4(a + 3) = 20$
5. $2.3n - 8 = 1.2n + 3$
6. $\frac{3}{8}y - 5 = \frac{5}{8}y - 3$
7. $6 + 2(x - 4) = 2(x - 1)$
8. $\frac{1}{3}(9b + 1) = b - 1$

9. MULTIPLE CHOICE For a project, a class is divided into two groups and each group has to make a video. Group A's video is 20 seconds less than twice the length of Group B's video. Group A's video is also 255 seconds longer than Group B's video. Which equation represents this information?
   A $2a + 20 = b + 255$
   B $2b - 20 = b + 255$
   C $20b - 255 = b + 20$
   D $a + 255 = b - 20$

For Exercises 10–12, define a variable and write an equation to find each number. Then solve.

10. Eight more than three times a number equals four less than the number.
11. The product of a number and five is twelve more than the number.

12. GEOMETRY The perimeter of the rectangle is 22 feet. Find the dimensions of the rectangle.

13. SHOPPING The cost of purchasing four shirts is at least $120. Write an inequality to describe this situation.

Write an inequality for each graph.

14. [Graph with two lines and a shaded region]
15. [Graph with a shaded region between two lines]
16. [Graph with a shaded region between two lines]

17. SALES The Cookie Factory has a fixed cost of $300 per month plus $0.45 for each cookie sold. Each cookie sells for $0.95. How many cookies must be sold during one month for the profit to be at least $100?

18. MULTIPLE CHOICE Danny earns $8.50 per hour working at a movie theater. Which inequality can be used to find how many hours he must work each week to earn at least $100 a week?
   F $8.50h < 100$
   H $8.50h \leq 100$
   G $8.50h > 100$
   J $8.50h \geq 100$

Solve each inequality and check your solution. Then graph the solution on a number line.

19. $-4 \geq p - 2$
20. $3x \geq 15$
21. $-42 < -0.6x$
22. $c - 3 \leq 4c + 9$
23. $7(3 - 2b) \geq 5b + 2$
24. $\frac{1}{2}(a + 4) > \frac{1}{4}(a - 8)$

25. MULTIPLE CHOICE The Lapeer Nature Club wants to raise at least $4000 for conservation. They have been given a $150 dollar donation and are selling canvas bags for $55 each to raise the rest of the money. Which inequality describes how many bags they need to sell in order to reach this goal?
   A $x \geq 35$
   B $x \leq 35$
   C $x \leq 70$
   D $x \geq 70$
Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. The cost, \( c \), of renting a moving truck can be found using the equation \( c = 20.75 + 31.50d \), where \( d \) is the number of days you rent the truck. What would be the total cost of renting a truck for 4 days?
   A $126.00  
   B $146.75  
   C $158.50  
   D $172.25

2. A music store surveyed 100 of its customers about their preferred styles of music. The results of the survey are shown in the survey.

<table>
<thead>
<tr>
<th>Favorite Style of Music</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Country</td>
<td>25</td>
</tr>
<tr>
<td>Rock</td>
<td>38</td>
</tr>
<tr>
<td>Jazz</td>
<td>18</td>
</tr>
<tr>
<td>Classical</td>
<td>12</td>
</tr>
<tr>
<td>Other</td>
<td>7</td>
</tr>
</tbody>
</table>

If the store only uses these data to order new CDs, what conclusion can be drawn from the data?
   F More than half of each order should be country and rock CDs.
   G More than half of each order should be rock CDs.
   H Only country, rock, and jazz CDs should be ordered.
   J About a fourth of each order should be classical music CDs.

3. GRIDDABLE Find the next term in the pattern below.
   1, 3, 7, 13, 21, 31, ...

4. A sequence of numbers was generated using the rule \( 3n - 1 \), where \( n \) represents a number’s position in the sequence. Which sequence fits this rule?
   A 1, 3, 5, 7, 9, ...
   B 1, 4, 7, 10, 13, ...
   C 2, 4, 6, 8, 10, ...
   D 2, 5, 8, 11, 14, ...

5. Which fraction is between \( \frac{3}{4} \) and \( \frac{4}{5} \)?
   F \( \frac{2}{3} \)
   G \( \frac{5}{7} \)
   H \( \frac{19}{25} \)
   J \( \frac{7}{8} \)

6. GRIDDABLE Colleen is using a punch recipe that calls for 12 ounces of fruit juice for every 40 ounces of lemon-lime soda. If she uses 60 ounces of lemon-lime soda, how many ounces of fruit juice will she need?

7. Robert, Isabelle, Michael, and Katrina are going to a football game. The total cost of the tickets is $231.75. Robert paid $60, Isabelle paid 20% of the total cost, Michael paid \( \frac{1}{4} \) of the total cost, and Katrina paid the rest. Who paid the greatest amount?
   A Robert
   B Isabelle
   C Michael
   D Katrina

8. A couch is on sale for 20% off the regular price of $480. How much money is discounted off the regular price?
   F $384
   G $362
   H $96
   J $84
9. Kelly’s deck has an area of 660 square feet.

What is the length of the deck if the width is 11 feet?
A 66 ft  B 60 ft  C 50 ft  D 45 ft

10. Mary Ann saved $56 when she purchased a television on clearance at an electronics store. If the sale price was 20% off the regular price, what was the regular price?
F $250  G $260  H $275  J $280

11. The cost, \( c \), of hiring a plumber can be found using the equation \( c = 75 + 40h \), where \( h \) is the number of hours the plumber worked. What would be the total cost of hiring a plumber for 3 hours?
A $40  B $75  C $195  D $120

12. The cost, \( c \), of renting a car can be found using the equation \( c = 50 + 0.10m \), where \( m \) is the number of miles you drive the car. What would be the total cost of renting a car and driving it 200 miles?
F $20  H $50  G $0.10  J $70

13. The cost, \( c \), of renting a tent site can be found using the equation \( c = 12.50 + 2.50p \), where \( p \) is the number of people you will have on the site. What would be the total cost of renting a tent site for 5 people?
A $12.50  C $25.00  B $15.00  D $65.00

If you get finished with the test before the end of the time allowed, go back and check your work.

14. Kevin earns a monthly salary of $1750. In addition to his salary, he receives a $250 bonus for every car that he sells. He wants to earn at least $3000 per month.

a. Write an inequality to represent this situation.
b. Solve the inequality that you found in part a.
c. What is the minimum number of cars he must sell?
d. How many cars will he have to sell if he wants to earn at least $4000 per month?